The Japanese Stagnation: an Assessment of the Productivity Slowdown Hypothesis.

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Abstract

The paper analyses the process of long-run growth in Japan over the period 1957-2001, using a common trends model, in order to assess whether the economic slowdown in Japan over the 1990s has been caused by a productivity slowdown. The paper finds empirical support for the neoclassical growth model: the long-run evolution of the economy may be related to two persistent shocks which bear the interpretation of productivity and labour supply shocks. Coherent with the simulation results of Hayashi and Prescott (2002), it is found that, over the 1990s, the potential output path has been characterised by both a reduction in level and slope. Since supply side reforms are still in their infancy, it is likely that the recovery of the Japanese economy is going to be a long lasting process.

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1. Introduction

Both demand and supply side explanations for the Japanese slowdown\(^1\) have been suggested in the literature, among which (a) inadequate fiscal and monetary policies to bring the economy out of the liquidity trap, (b) depressed investment due to over-investment during the bubble period of the late 1980s and early 1990s and problems with financial intermediation following the bursting of the bubble, and (c) the reduction in potential output growth determined by a productivity slowdown and demographic effects.

In the paper we focus on the productivity slowdown hypothesis. As argued by Hayashi and Prescott (2002), the economic slowdown in Japan over the 1990s may be accounted by a fall in total factor productivity growth, coupled with a reduction of the workweek length due to revisions on labour laws. According to the standard general equilibrium growth model, the first shock would have reduced the slope of the steady state growth path and increased the steady state capital/output ratio. On the other hand, the latter shock would have shifted the steady state growth path down. Calibration of the model supports the explanation provided by the authors. The aim of the paper is to complement the simulation analysis of Hayashi and Prescott (2002) with additional empirical evidence. In the framework of a common trends model, we aim to identify the growth engines underlying long-run growth in Japan, to obtain an estimate of potential output, and to assess whether the potential output path has been subject to a downward shift in level and slope, as claimed by these authors.

The main findings of the paper are as follows. Firstly, we find empirical support for the neoclassical growth model and its prediction of balanced growth. The

\(^1\)After rapid growth at an average annual rate of 3.7% between 1981 and 1992, Japan has entered a period of stagnation still affecting the economy, with output growing at an average annual rate of about 1%, rising unemployment and negative inflation (at an average rate of about -1% since 1998). In fact, apart from rapid growth in 1996 (3.3%), output decelerated in 1997 (1.9%), to turn negative in 1998 (-1.1%) and 2001 (-0.2%) and moderately positive in 1999 (0.8%) and 2000 (1.5%). The recession started in 1998 is the most severe experienced by Japan since the postwar period, and still there are no signs of full recovery.
long-run growth process may be related to productivity and labour supply shocks. Interestingly, the productivity shock explains the bulk of output fluctuations both in the short and long-run, validating the theoretical framework employed in the analysis. Secondly, we find evidence of a marked reduction in both the level and slope of the potential output path. This finding is fully coherent with the simulation results of Hayashi and Prescott (2002) and with the supply side explanation of the economic slowdown in Japan. Since supply side reforms in Japan are still in their infancy, it is likely that the recovery of the Japanese economy is going to be a long lasting process.

The paper is organised as follows. In section two we discuss the supply side explanation of the economic slowdown in Japan. In section three we present the theoretical framework and in section four the econometric methodology. In section five we present the empirical results. Finally, in section six we conclude.

2. The Japanese economic slowdown: the supply side explanation

A recent explanation for the economic slowdown in Japan over the 1990s points to a reduction in potential output growth determined by a productivity slowdown and demographic effects. Hayashi and Prescott (2002) have documented a fall in total factor productivity growth, coupled with a reduction of the workweek length from 44 hours to 40 hours between 1988 and 1993 due to revisions on labour laws. According to the standard general equilibrium growth model, the first shock would have reduced the slope of the steady state growth path and increased the steady state capital/output ratio. On the other hand, the latter shock would have shifted the steady state growth path down. Calibration of the model suggests that the stagnation of the Japanese economy over the 1990s is what the model predicts. Hayashi and Prescott (2002) have pointed to policies which subsidized inefficient firms and declining industries as causal factors of the productivity slowdown, given the large share of total output produced by inefficient firms in Japan over the 1990s. In addition, such policies, by discouraging investment, may impede the introduction of new and more productive technologies.

A productivity slowdown explanation has also been proposed by Yamaguchi (1999). According to the author, the productivity slowdown may be related to firms’ inability to adapt to changes and globalization. Three explanations along these lines are proposed. Firstly, non performing loans would have lead banks to restrain from providing the capital necessary for firms to adapt to changes through
new investments.  

Secondly, laws, regulations and the tax structure would have prevented firms to benefit fully of the revolution in information technologies.  

Thirdly, the rigid labour market would have prevented the necessary flow of labour from declining industries to ascendant industries and from inefficient firms to efficient firms.

On the other hand, in demand side explanations as Krugman (1998), the productivity slowdown can be seen as an impulse mechanism, with other forces being the causal factor of the stagnation, i.e. the implementation of wrong economic policies to bring the economy out of the liquidity trap. In fact, while high investments during the 1980s could be explained by the rapid growth in output and potential output over the same period, the fall in total factor productivity and the effects of a shrinking labour force would have caused a fall in potential output,

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2 A thorough account of the banking crisis in Japan has been provided by Hoshi and Kashyap (1999) and Kanaya and Woo (2000). The key explanatory factors have been identified in the following. Firstly, excessive credit expansion during the 1980s would have occurred as a consequence of collateral backed loans, rather than cash flows based assessment of customer solvency, as asset prices climbed. In addition, financial deregulation would have lead to a reduction in interest rates margins and the expansion of credit to riskier segments of the market, as consumer lending, real estate industry and small and medium enterprises. After the collapse of the bubble economy, banks and financial institutions would have started facing solvency problems due to non performing loans and the reduction in the value of collaterals and equity holdings. As a consequence of the downgrading of Japanese banks in 1992, marginal costs of funding would have increased, leading large firms to seek alternative sources of funding directly in the bonds market, and banks to increase lending to small and medium sized enterprises. Banks also would have revised credit evaluation criteria, making them more stringent and based on cash flows projections. The overall effect would have been an impaired banks' ability to lend and firms' ability to borrow, leading to a reduction of loans, investment, and output.

3 It is not clear whether the “ICT revolution” has lead to productivity increases even in the countries which have fully implemented the new developments, as for instance the US. In fact, some critical authors as Gordon (2000b) have suggested that the structural increase in productivity in the period 1995-1999 in the US economy, computed relatively to the period 1972-1995, has been generated in the ICT sector, and induced positive spillovers in the durable sector, while the remaining sectors would have only experienced a reduction in productivity. More recently, Gordon (2003) has suggested that most of the productivity growth revival in the 1990s may be associated with non-ICT investments and organisational innovations, leaving future productivity dynamics highly uncertain. A different view has been expressed by Feldstein (2003), pointing to ICT determined productivity gains continuing also in the future, possibly at increasing rates. Yet, according to the author, a necessary condition would be the presence of an institutional framework that provides incentives for managers to undertake the risk of implementing the innovations associated with the information technologies. In this respect, perspectives for Japan would be much more uncertain.
leading to a contraction in investment demand, a saving-investment imbalance, a fall in nominal interest rates and the emergence of the liquidity trap. Despite these facts, monetary policy would have been mostly carried out in a conventional manner, i.e. by operating on the nominal overnight rate, and overly restrictive, with outright purchases of long term bonds only of the size necessary to match money demand developments. A more aggressive purchase of long term bonds has been also suggested by Goodfriend (2000) and Posen (2002), in order to stimulate investment demand through Tobin’s q and financial accelerator effects and the direct financing of the corporate sector and government-affiliated financial institutions, and/or consumption through wealth effects and the underwriting of government bonds in compensation for a tax cut. Also the fiscal instrument, the traditional solution to the liquidity trap, would have not been used appropriately or would have failed to determine the expected improvements. In fact, while evidence provided by Kuttner and Posen (2001, 2002) suggests that fiscal multipliers were larger than one, fiscal policy in Japan over the 1990s has tended to be contractionary, while Krugman (1998) has argued in favour of significant Ricardian effects.

Hayashi and Prescott (2002) have argued against a demand (investment) based explanation for the Japanese stagnation. In fact, neither the liquidity trap nor the credit crunch hypothesis seem to be valid explanations, since the investment to output ratio would have not shown a decline during the 1990s relatively to

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4In Japan short term rates were reduced gradually between 1990 and 1995 to levels close to zero, and maintained at fractional levels thereafter. The zero interest rate policy was started in 1999, reversed in August 2000 and reversed again in March 2001, coupled with a quantitative easing aimed at increasing banks reserves. According to BoJ economists (Okina, 1999; Mori et al., 2001), monetary policy over the 1990s has been accommodative, while many authors have contended that monetary policy has been restrictive, pointing to the need of non hortodox measures in order to allow base money to growth at higher rates. For instance, McCallum (2001a) has suggested that an appropriate annual rate of growth for base money would have been close to 12% for a decade, rather than the observed 5%.

5Other non orthodox policy measures have been proposed by McCallum (2000, 2001a,b), Svensson (2000), and Meltzer (1999). See Fujiki et al. (2001) for a detailed account of the non hortodox policy measures suggested in the literature.

6In 1992 and 1993 public expenditure increased of about 2% of GDP. In 1994 a temporary income tax reduction of about 0.6% of GDP, reversed in 1997, was introduced. In 1997 both the consumption tax and contributions to social security were increased. Overall the tax burden increased of about 2% of GDP. In 1998 public works expenditure increased again, coupled with a permanent reduction in the personal income tax, for a total of 4% of GDP. In 1999 further fiscal expansions were announced but not implemented, and no other expansionary policies were undertaken in 2000 and 2001.
the 1980s, remaining close to 15%, apart from the increase in the late 1980s and early 1990s. According to the authors, therefore, firms would have managed to find alternative sources of investment to bank loans, for instance by drawing down the land and financial assets accumulated during the 1980s.7 Hence, the fall in productivity could not just be considered as the impulse mechanism of the economic slowdown, but, with the negative labour supply shock, its actual causal factor.

If the productivity slowdown explanation of the economic stagnation is correct, then, as documented by Roubini (1996), several reforms would be needed to improve the supply side of the Japanese economy and foster potential output growth. Firstly, financial markets liberalization and financial restructuring of banks are needed to avoid credit constraints to be binding now or in the future. A second set of reforms concerns the improvement of supply conditions through a deregulation process of the economy which promotes competition, efficiency, productivity, technological innovations and reduces labour costs. Finally, administrative and political reforms are needed to ensure a more efficient working of the public sector, and educational reforms are necessary to promote basic research, rather than applied research, in universities and technological innovation.

A recent work of the Japanese Centre for Economic Research (2002) reports forecasts for the Japanese economy up to 2010, suggesting that a sustained recovery in the Japanese economy will be achieved only starting from 2007, conditional to some key supply side reforms be carried out. In particular, an acceleration of bad loans clean up, a tax reduction centered on corporate taxation (reduced taxes on R&D and investment), and an increase in the labour force participation of women. These reforms would be coupled by an improvement in the fiscal balance in the medium term, achieved through a reduction of government spending (public work and health care spending, the number of civil service employees) and an increase in the taxable base and in the consumption tax. The exercise is inter-

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7It should be noted that in the literature there is substantial agreement concerning the reduction of credit availability during the 1990s, particularly from 1997 onwards (Woo, 1999; Motonishi and Yoshikawa, 1999). However, there is much less agreement concerning the effects that the reduction in bank loans actually produced, i.e. whether it lead to a fall in investment and output. According to Kuttner and Posen (2001), the contraction in bank lending would have affected small and medium sized enterprises, since large firms were able to finance themselves going directly to capital markets (see also Sekine, 1999; Kanaya and Woo, 2000). Evidence in favour of a negative impact on investment have also been provided by Kwon (1998), Bayoumi (1999), Ogakawa and Suzuki (1998), Ogakawa et al. (1996), Morsink and Bayoumi (1999), and Rendu (1999).
esting, and suggests that the supply side explanation of the economic slowdown is gaining increasing consensus.

3. Theoretical framework

The theoretical framework of the paper is the neoclassical growth model as in Hayashi and Prescott (2002). Despite the theoretical advances in the theory of economic growth made since the mid 1980s, the empirical evidence provided during a decade of studies has not allowed for a clear-cut rejection of the key features of the neoclassical theory of growth, as exogenous growth, constant returns to scale and diminishing returns to the accumulated factor. Evidence in support of a stochastic version of the Solow model and against a stochastic AK model have been recently provided by Binder and Pesaran (1999), suggesting that the neoclassical growth model is far from being dismissed by the data. This latter finding provides additional evidence to the seminal results of King et al. (1991) and Neusser (1991), supporting the stochastic neoclassical model and its prediction of balanced growth, i.e. the stationarity of the great ratios. Implicit support for the neoclassical growth model can also be found in the work of Gordon (2000a,b), namely in the association of productivity advances with the introduction of some key inventions, which brought deep improvements in the quality of life, and originated independently of economic forces.8

The model predicts that two exogenous engines drive the process of growth in the long-run, namely productivity and labour supply growth, and that the key variables in the model, i.e. output, consumption, investment, and the capital stock grow at the same rate, determined by the two growth engines. The model also predicts that the same variables in per capita term growth at a rate determined by the productivity improvement. When productivity and labour supply follow a stochastic process rather than a deterministic process, the model predicts that the evolution over time of the key variables in levels is determined by two common

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8 Gordon (2000a) identifies four major groups of inventions in the late nineteenth and early twentieth century: electric light and motors, internal combustion engines, petroleum and processes which rearrange molecules (petrochemicals, pharmaceuticals) and communications innovations. Interestingly, according to Gordon (2000b), the invention of the Internet in the mid-1990s was a much less important invention, unlikely to produce the productivity advances expected by many authors. Even the invention of the computer would have already exercised its benefits, due to its strong diminishing returns, determined by the fixed amount of time and brainpower of users.
stochastic trends, and that the great ratios, i.e. consumption-income, investment-income, capital-income ratios, are stationary processes. This implies the existence of cointegration among the variables in the model.

The basic structure of the balanced path solution of the standard neoclassical growth model is sketched below. Two non stationary stochastic processes drive the evolution of the variables in levels along the balanced path, namely an exogenous technology variable

$$a_t = \gamma_1 \theta_t$$  \hspace{1cm} (3.1)

$$\theta_t = \mu_\theta + \theta_{t-1} + \nu_{\theta,t},$$  \hspace{1cm} (3.2)

$$\nu_{\theta,t} \sim \text{i.i.d}(0, \sigma_{\nu_{\theta}}^2),$$

and an exogenous labour supply variable

$$e^*_t = \gamma_2 \xi_t,$$  \hspace{1cm} (3.3)

$$\xi_t = \mu_\xi + \xi_{t-1} + \nu_{\xi,t},$$  \hspace{1cm} (3.4)

$$\nu_{\xi,t} \sim \text{i.i.d}(0, \sigma_{\nu_{\xi}}^2).$$

By writing the deviation from the log (linearised) steady state solution of the model as

$$y_t - a_t - e^*_t = y^* + \varepsilon_{y,t}$$

$$c_t - a_t - e^*_t = c^* + \varepsilon_{c,t}$$

$$i_t - a_t - e^*_t = i^* + \varepsilon_{i,t}$$

$$k_t - a_t - e^*_t = k^* + \varepsilon_{k,t},$$  \hspace{1cm} (3.5)

where \textquotedblleft**\textquotedblright\textquotedblright denotes the steady state per capita value of the variables expressed in efficiency units, \(\varepsilon_{j,t} \hspace{0.5cm} j = y, c, i, k\) are i.i.d. disturbance processes, the following equations for the variables in levels can be obtained

$$y_t = y^* + \gamma_1 \theta_t + \gamma_2 \xi_t + \varepsilon_{y,t},$$
\[ c_t = c^* + \gamma_1 \theta_t + \gamma_2 \xi_t + \varepsilon_{c,t}, \]
\[ i_t = i^* + \gamma_1 \theta_t + \gamma_2 \xi_t + \varepsilon_{i,t}, \]
\[ k_t = k^* + \gamma_1 \theta_t + \gamma_2 \xi_t + \varepsilon_{k,t}. \quad (3.6) \]

It is immediate to verify that the model predicts that the great ratios are stationary processes

\[ c_t - y_t = c^* - y^* + \varepsilon_{c,t} - \varepsilon_{y,t}, \]
\[ i_t - y_t = i^* - y^* + \varepsilon_{i,t} - \varepsilon_{y,t}, \]
\[ k_t - y_t = k^* - y^* + \varepsilon_{k,t} - \varepsilon_{y,t}, \quad (3.7) \]

yielding three plausible candidate long run relationships.

A fourth candidate long-run relationship is provided by the labour demand equation.

\[ e^d_t = \lambda y_t - \eta w_t + \varepsilon_{e,t}, \quad (3.8) \]
\[ \varepsilon_{e,t} \sim i.i.d(0, \sigma^2_{\varepsilon_e}), \quad (3.9) \]

where \( w_t \) is the log real wage, and \( \lambda \) and \( \eta \) are the output and wage elasticities. A competitive labour market would imply \( e^d_t = e^*_t \), and \( \lambda = \eta = 1 \), i.e. a stationary wage share

\[ w_t + e_t - y_t = \varepsilon_{e,t}, \quad (3.10) \]

or a homogeneous relationship between the real wage and productivity (\( a_t \))

\[ w_t = y^* + \gamma_1 \theta_t + \varepsilon_{y,t} + \varepsilon_{e,t}. \quad (3.11) \]

Given the six variables in the model (consumption, investment, the stock of capital, output, employment, and the real wage) and the two common trends (productivity and labour supply), the four above mentioned stationary relationships are necessary and sufficient to close the system.
From Tobin’s q theory it is possible to postulate a relationship between the market valuation of the stock of capital \((f_t)\) and its replacement cost \((k_t)\). We have in fact

\[ q_t = f_t - k_t, \]  

(3.12)

that is

\[ f_t = q_t + k_t + \varepsilon_{q,t}, \]  

(3.13)

where \(q_t = q + \varepsilon_{q,t}\), which then enables to rewrite the stationary capital-output ratio in terms of the stock market index \(f_t\), yielding

\[ f_t - y_t = q + k^* - y^* + \varepsilon_{k,t} - \varepsilon_{y,t} + \varepsilon_{q,t}, \]  

(3.14)

and

\[ f_t = q + k^* + \gamma_1 \theta_t + \gamma_2 \xi_t + \varepsilon_{k,t} + \varepsilon_{q,t}. \]  

(3.15)

According to Hall (2001), the capital stock measured by the value of equities is a comprehensive aggregate, reflecting both tangible and intangible assets firms employ in production. It can be argued that this measure of capital is closer to the aggregate capital stock postulated in modern growth theory, reflecting physical capital, knowledge and human capital. The modelling framework we follow, therefore, is closer in spirit to Mankiw et al. (1992), rather than to the original model of Solow (1956).

Neglecting constants, the long-run evolution of the variables in per capita terms can then be summarised by the following system of equations

\[
\begin{bmatrix}
y_t - e_t \\
c_t - e_t \\
i_t - e_t \\
f_t - e_t \\
w_t \\
e_t
\end{bmatrix} =
\begin{bmatrix}
\gamma_1 \\
\gamma_1 \\
\gamma_1 \\
\gamma_1 \\
\gamma_1 \\
\gamma_1
\end{bmatrix}
\begin{bmatrix}
\theta_t \\
\xi_t
\end{bmatrix},
\]  

(3.16)

which shows how per capita income, consumption, investment, the market value of the stock of capital, and the real wage are determined by the productivity trend, while the long-run dynamics of employment are determined by labour supply conditions.
4. Econometric methodology

The vector equilibrium correction model (VECM) can be written as:

$$\Pi^\ast(L) \Delta x_t = \nu + \Pi x_{t-1} + \varepsilon_t$$

(4.1)

where \(x_t\) is a vector of \(n\) \(I(1)\) cointegrated variables of interest, \(\varepsilon_t \sim NID(0, \Sigma)\), \(\Pi(L) = I_n - \sum_{i=1}^{p} \Pi_i L^i\), \(\Pi = -\Pi(1)\), \(\Pi^\ast(L) = I_n - \sum_{i=1}^{p-1} \Pi^\ast_i L^i\) and \(\Pi^\ast_i = -\sum_{j=i+1}^{p} \Pi_j\) \((i = 1, \ldots, p-1)\).

If there are \(0 < r < n\) cointegration relationships among the variables, \(\Pi(1)\) is of reduced rank \(r\) and can be expressed as the product of two \((n \times r)\) matrices: \(\Pi(1) = \alpha \beta\), where \(\beta\) contains the cointegrating vectors, such that \(\beta' x_t\) are stationary linear combinations of the \(I(1)\) variables, and \(\alpha\) is the matrix of factor loadings.

The Wold vector moving average representation (VMA) of the cointegrated system can be obtained following Mellander, Vredin and Warne (1992) and Warne (1993).

4.0.1. RVAR and common trends representations

The restricted VAR representation (RVAR) can be written as

$$B(L) Y_t = \theta + \eta_t$$

(4.2)

where \(B(L) = T [\Pi^\ast(L) T^{-1} D(L) + \alpha^* L]\), \(Y_t = D_\perp(L) T x_t\), \(T = [\beta'_ \beta]'\), \(\alpha^* = \begin{bmatrix} 0 & \alpha \end{bmatrix}\) and \(D(L)\) and \(D_\perp(L)\) are polynomial matrices defined by

$$D(L) = \begin{bmatrix} I_k & 0 \\ 0 & (1-L) I_r \end{bmatrix}, D_\perp(L) = \begin{bmatrix} (1-L) I_k & 0 \\ 0 & I_r \end{bmatrix}.$$

The RVAR representation has the property that all the variables are stationary, either because they are expressed in first differences or as stationary linear relations. Then, the RVAR can be inverted to obtain the common trends representation of Stock and Watson (1988), which, in structural form, can be written as

$$x_t = x_0 + \mu t + \Gamma(1) \sum_{j=0}^{t-1} \varphi_{t-j} + \Gamma^\ast(L) \varphi_t$$

$$= x_0 + \mu t + \Gamma_g \sum_{j=0}^{t-1} \psi_{t-j} + \Gamma^\ast(L) \varphi_t,$$

(4.3)
where $\varphi_t \equiv \left[ \psi_t \quad \upsilon_t \right]' \sim I.I.D.(0, I_n)$, with $\psi_t$ and $\upsilon_t$ subvectors of structural shocks of $k$ and $r$ elements respectively, $\varepsilon_t = \Gamma_0 \varphi_t$, and $\Gamma(1) = \sum_{j=0}^{\infty} \Gamma_j$, $\Gamma^*(L) = \sum_{j=0}^{\infty} \Gamma_j^* L^j$, $\Gamma^*_j = - \sum_{i=j+1}^{\infty} \Gamma_i$, where $\Gamma_j$ are matrices of parameters in the structural Wold vector moving average (VMA) representation. The existence of $r$ cointegrating vectors implies that the long-run matrix $\Gamma(1)$ has rank $n - r \equiv k$ and $\beta \Gamma(1) = 0$.

In order to identify the elements of $\psi_t$ as the permanent shocks and the elements of $\upsilon_t$ as transitory disturbances, only the disturbances in $\psi_t$ should be allowed to have long-run effects on (at least some of) the variables in $x_t$. Hence, $\Gamma(1) = \left[ \begin{array}{cc} \Gamma_g & 0 \end{array} \right]$, being $\Gamma_g$ a submatrix of dimension $n \times k$.

**Identification of the shocks** To identify the common trends model it is necessary to find a matrix $\Gamma_0$, such that it can be uniquely determined from the parameters of the VECM model, where the variance covariance matrix of $\Gamma_0^{-1} \varepsilon_t = \varphi_t$ is diagonal with non zero entries, and the long-run impact matrix is $\Gamma(1) = \left[ \begin{array}{cc} \Gamma_g & 0 \end{array} \right]$.

By rewriting the mapping from the reduced form disturbances to the structural disturbances as

$$\Gamma_0^{-1} \varepsilon_t = \varphi_t \Leftrightarrow \left[ \begin{array}{c} G \\ H \end{array} \right] \varepsilon_t = \left[ \begin{array}{c} \psi_t \\ \upsilon_t \end{array} \right],$$

it can be noticed that through the $(k \times n)$ matrix $G$ the reduced form disturbances are mapped into permanent disturbances, and through the $(r \times n)$ matrix $H$ the reduced form disturbances are mapped into transitory disturbances.

Following Warne (1993), the matrix $G$ can be estimated as

$$G = (\Gamma_g' \Gamma_g)^{-1} \Gamma_g' \Sigma(1), \quad (4.4)$$

where $\Sigma(1)$ is the long-run impact matrix in the reduced form Wold VMA representation.

To estimate the $(n \times k)$ matrix $\Gamma_g$, we need (at least) $nk$ restrictions on its elements. Cointegration implies

$$\beta \Gamma_g = 0, \quad (4.5)$$

yielding $kr$ linear restrictions. Additional $k(k + 1)/2$ restrictions on the elements of $\Gamma_g$ are provided by assuming $E(\psi_t \psi_t') = E(G \Sigma G') = I_k$. That is, $k(k + 1)/2$ restrictions are given by

$$\Sigma(1) \Sigma(1)' = \Gamma_g' \Gamma_g, \quad (4.6)$$

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since \( C(1) \) and \( \Gamma_g \) have reduced rank \( k \). The remaining \( k(k-1)/2 \) restrictions needed for (exact) identification of \( \Gamma_g \) have to be derived from economic theory.

To estimate the \((r \times n)\) matrix \( H \), we need (at least) \( nr \) restrictions on its elements. It can be noticed that from the orthogonality condition \( E[\psi_t \nu'_t] = 0 \) we have

\[
E(G\varepsilon_t \varepsilon'_t H') = G\Sigma H' = 0,
\]

that is

\[
(\Gamma'_g \Gamma_g)^{-1} \Gamma'_g C(1) \Sigma H' = 0.
\]

Hence, reminding that \( C(1)\alpha = 0 \), a possible solution for \( H \) takes the form

\[
H = Q^{-1} \zeta \Sigma^{-1},
\]

where \( \zeta = \alpha (U\alpha)^{-1} \), \( U \) is a matrix chosen in such a way that \( U\alpha \) is non singular, and the \((r \times r)\) matrix \( Q \) is such that \( E[\nu_t \nu'_t] = I_r \). In practice the matrix \( Q \) can be obtained from the Choleski decomposition of \((\zeta' \Sigma^{-1} \zeta)^{-1}\). The estimation of \( H \) requires the imposition of \( r(r-1)/2 \) additional restrictions on the \((r \times r)\) matrix \( \zeta \), since the remaining \( kr + r(r+1)/2 \) restrictions necessary for exact identification are provided by the orthogonality conditions \( E[\psi_t \nu'_t] = 0 \) and \( E[\nu_t \nu'_t] = I_r \).

By noting that \( \Sigma = \Gamma_0 \Gamma'_0 \), we have that \( \Gamma_0 = \Sigma (\Gamma'_0)^{-1} = \begin{bmatrix} \Sigma G' & \Sigma H' \end{bmatrix} \). Thus, the contemporaneous impact matrix can be written as

\[
\Gamma_0 = \begin{bmatrix} \Sigma C(1)' \Gamma_g (\Gamma'_g \Gamma_g)^{-1} \zeta (Q^{-1})' \end{bmatrix}. \tag{4.9}
\]

From the moving average representation impulse response functions and forecast error variance decompositions can be calculated with respect to permanent and transitory innovations.

5. Empirical results

The national accounts, employment, stock market and wage data are from the IMF Financial Statistics Yearbook and cover the period 1957:1-2001:4.

5.1. Integration and cointegration properties of the data

Standard ADF tests have been employed to evaluate the integration properties of the data. Coherent with the theoretical framework, output, consumption, investment and stock market data have been expressed in percapita terms. All the
series have been log transformed and, apart from the employment series, deflated by the GDP deflator. Since all the series in levels show I(1) properties, cointegration analysis has been performed by means of the Johansen (1988) estimator. The order of the VAR model has been determined by means of both information criteria and specification tests. The Schwarz-Bayes and the Hannan-Quinn criteria point to a VAR(1) model, while the Akaike information criterion selects a VAR(5) model. Since the VAR(1) model does not show evidence of serial correlation at the 1% significance level, apart from the real wage equation, the most parsimonious specification has been selected.\(^9\) Four cointegrating vectors can be selected according to the Trace statistic, pointing to the existence of two common stochastic trends driving the six variables in the system (Table 1). Coherent with the theoretical framework, the former three identified cointegrating vectors can be interpreted as the consumption-income, investment-income and capital-income ratios, while the fourth cointegrating vector can be interpreted as a labour demand equation, which is however not consistent with the theoretical restrictions. The implied labour demand equation can in fact be written as

\[ e_t = 0.64Y_t - 0.44w_t, \]

pointing to a rigid response of employment to output (\(Y\)) and the real wage (\(w\)).\(^{10}\)

5.2. Short run dynamics

The estimated loading matrix is reported in Table 1. As is shown in the table, none of the variables can be regarded as weakly exogenous relative to the long-run parameters. According to the estimated parameters excess real wages are corrected by a reduction in both wages and employment, although the speed of adjustment is low (-0.03). This finding is coherent with the well known rigidities characterising the Japanese labour market. Also the stock market, investment and output react negatively to excess real wages, showing however a higher speed of adjustment. Consumption, investment and the stock market react negatively to their own disequilibrium, finding which supports the identification restrictions

\(^9\)The results of the stationarity and specification analysis are available upon request to the author.
\(^{10}\)The cointegration properties of the VAR(5) model have been investigated as a robustness check. The identified cointegration space is also validated in this latter case. The p-values of the Likelihood-Ratio test are 0.064 and 0.151.
imposed on the cointegration space. Consumption and investment show a similar speed of adjustment (-0.15, -0.12), while the stock market reacts slowly to its own disequilibrium. Interestingly, the stock market reacts negatively to excess investment and consumption, showing a very high speed of adjustment (-0.87 and -0.69): more than half of the disequilibrium is covered within a quarter. Moreover, consumption reacts negatively to excess investment, while investment reacts negatively to excess consumption. Finally, only investment reacts (positively) to the stock market disequilibrium, pointing to significant, albeit weak, Tobin’s q effects. This latter result and the non significant reaction of consumption to the stock market disequilibrium cast some doubts on the effectiveness of the policy remedies proposed by Goodfriend (2000) and Posen (2002) to revitalise aggregate demand. Moreover, the fact that consumption and investment, rather than output, correct relatively to their own disequilibria point to the presence of neoclassical features in the Japanese economy, suggesting that aggregate supply influences aggregate demand also in the short-run. This finding provides implicit support for the productivity slowdown explanation of the economic slump.

5.3. The common trends model

The neoclassical growth model predicts that two exogenous growth engines, namely technical progress and labour supply, drive the six variables in the system in the long-run. As already noted, the results of the cointegration analysis are coherent with this prediction, pointing to four cointegrating vectors and two common trends. In order to identify the structural shocks as productivity and labour supply shocks a single exclusion restriction on the factor loading matrix is required. Since the output, consumption, investment, and stock market series have been expressed in per capita terms, the exactly identifying restriction requires that the labour supply shock does not affect per capita output in the long-run. Given the structure of the cointegration space, this restriction also implies that the labour supply shock does not affect per capita consumption, investment and the stock market in the long-run. Since there are only two permanent shocks, the exclusion restriction implies that the shock which affects per capita output in the long-run may be interpreted as the productivity shock, conditional to coherent responses of the variables of the model. Moreover, since both productivity and employment are significant variables in the long-run labour demand equation, entering with positive and negative coefficients, respectively, one should expect that both shocks will exercise a long-run impact on the real wage, with productivity ad-
vances leading to increases in the real wage and labour supply expansions leading to a reduction in the real wage. By normalising the labour demand equation relatively to employment, it also follows that technological progress should lead to an increase in employment, shifting the labour demand equation outwards. Finally, the labour supply shock should affect employment positively, and explain an important proportion of its variability in the long-run.

The identification of the CT model would require the imposition of six additional exclusion restrictions on the impact matrix ($\Gamma_0$) in the VMA representation. Since the focus of the study is on the long-run properties of the system and the identification of the permanent shocks is independent of the identification of the transitory shocks, we do not discuss this latter issue further in the paper.

The common trends representation of the variables in levels is then the following:

\[
\begin{pmatrix}
y - e \\
e - ec \\
i - ei \\
f - ef \\
w \\
e
\end{pmatrix}_t = \begin{pmatrix}
y - e \\
e - ec \\
i - ei \\
f - ef \\
w \\
e
\end{pmatrix}_0 + \begin{pmatrix}
\gamma_{1,1} & 0 \\
\gamma_{2,1} & 0 \\
\gamma_{3,1} & 0 \\
\gamma_{4,1} & 0 \\
\gamma_{5,1} & \gamma_{5,2} \\
\gamma_{6,1} & \gamma_{6,2}
\end{pmatrix} \begin{pmatrix}
\tau_\theta \\
\tau_\xi
\end{pmatrix}_t + \Gamma^* (L) \begin{pmatrix}
\psi_\theta \\
\psi_\xi \\
v_1 \\
v_2 \\
v_3 \\
v_4
\end{pmatrix}_t.
\]

The estimated factor loading matrix $\Gamma_g$ is reported in Table 2, while the forecast error variance decomposition of the variables is reported in Table 3. Standard errors for the impulse response functions have been computed by bootstrap simulation, with 1000 replications.

### 5.4. Forecast error variance decomposition

As shown in Table 3, the forecast error variance decomposition yields some interesting results. Firstly, the productivity shock explains the bulk of per capita output and consumption variance both in the short-run and in the long-run. In
fact, already at the one year horizon the productivity shock explains about 70% of variance, with the remaining proportion being explained by the transitory shocks (20%). The explained proportion of variance increases with the time horizon and is close to 95% within ten years. Different conclusions can be drawn for the other series. In particular, per capita investment is largely explained by the labour supply shock in the short term (40%), with productivity and the transitory shocks contributing in equal proportions (30%). However, the productivity shock becomes the main determinant of investment fluctuations already within three years (50%), explaining about 86% of investment variability within ten years. On the other hand, the transitory shocks explain the bulk of stock market variability in the short-medium term, with productivity being the only determinant of stock market fluctuations in the very long-run ($\infty$). Similar conclusions can be drawn for the real wage, although the bulk of fluctuations is explained by the productivity shock already within ten years (60%). Finally, while transitory shocks explain employment variability in the short term, in the medium-long term both productivity and the labour supply shocks contribute in equal proportions (50%) to fluctuations.11

5.5. Impulse response analysis

In Figures 1 and 2 we report the impulse response functions to the permanent shocks (unitary impulses).12 As shown in Figure 1, the response of the per capita variables to the first permanent shock are coherent with the suggested interpretation (productivity shock), leading to a progressive increase in all of them, apart from the per capita real stock market index. In fact, while for output, consumption and investment the effects of the shocks cumulate monotonically over time, for the stock market index the response path is more complex. Coherent with the identified cointegration space, a unitary productivity shock leads to a 5% increase

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11 The negligible contribution of transitory shocks to the explanation of short-run output dynamics contrasts with the results of King et al. (1991), Morana (2002) and DeLoach and Rasche (1998) for the US economy, Dhar et al. (2000) for the UK economy, and Cassola and Morana (2002) for the euro area. It is however consistent with the results of the short-run dynamics analysis carried out in the previous section and with the simulation results of Hayashi and Prescott (2002), which support the use of the neoclassical growth model to explain the economic slowdown in Japan over the 1990s. Also the negligible contribution of productivity shocks to the explanation of short-run stock market dynamics contrasts with results of Morana (2002) and Cassola and Morana (2002), although the sample period investigated is different.

12 To recover the impulse responses for the variables in levels, the response of the per capita variables should be adjusted for the response in employment.
in per capita output, consumption, investment and stock prices in the very long-run. Interestingly, the transition to the steady state seems to be very long lasting, requiring about 40 years to be accomplished. On the other hand, the long-run impact on the real wage is about +6%, while the increase in employment is about 1%. Technical progress therefore has lead to an expansion of employment over the time period considered, shifting labour demand outwards. This may also explain the excess response of real wages to the productivity shock.

By construction the labour supply shock does not exercise any long-run impact on the per capita variables (Figure 2).\textsuperscript{13} The adjustment path is however interesting, with output, consumption and investment declining over time, to converge back to the baseline within 40 years. On the other hand, the per capita stock market index is positively affected by the shock (+2% within six years), albeit temporarily. The interpretation of the second permanent shock as a labour supply shock is supported by the opposite long-run impact exercised on employment and the real wage, with the former variable increasing of about 1% and the latter variable falling of about 0.8%.

5.6. Permanent output dynamics

The common trends model is a natural framework to derive a permanent transitory decomposition of the GDP level. In particular, we are interested in the permanent component of GDP, which may be regarded as a measure of potential output. From the common trends model estimated on the per capita variables, the potential output estimate may be computed as $Y_p^t = y_p^t + e_p^t$, where

$$y_p^t = y_0 + \mu_y t + \Gamma_y (1)^{t-1} \sum_{j=0}^{t-1} \varphi_{t-j}$$

$$e_p^t = e_0 + \mu_e t + \Gamma_e (1)^{t-1} \sum_{j=0}^{t-1} \varphi_{t-j}$$

and then adjusting the transitory component $Y_t - Y_t^p$ in order to have a zero mean.

\textsuperscript{13}In the neoclassical growth model the variables in levels (output, consumption, investment, capital stock) increase at the rate determined by both productivity and labour supply. Hence, per capita variables grow at the rate determined by productivity advances only. The long-run response of the variables in levels to the labour supply shock is given by the long-run response of employment to the same shock (+1%), yielding a 1% increase in output, investment, stock market prices and consumption.
The estimated permanent component of the GDP level is plotted in Figure 3. As can be noted from the plot, the 1990s have been characterised by both a fall in the level and the slope of the potential output path. In particular, over the period 1990-1994 potential output has decreased steadily, while starting from 1995 potential output has started to increase again, albeit at a reduced rate. This findings are fully coherent with the results of Hayashi and Prescott (2002), and support the supply side explanation of the economic downturn in Japan. Hence, appropriate policy measures should aim at a fast recovery of productivity (see Roubini, 1996). Since supply side reforms are still in their infancy, the recovery of the Japanese economy is likely to be a long lasting process.

6. Conclusions

In this paper we have analysed the process of long-run growth in Japan over the period 1957-2001, using a common trends model, in order to assess whether the productivity explanation of the economic slowdown in Japan over the 1990s is of empirical relevance. We have found that the neoclassical growth model seems to be an appropriate theoretical framework to describe the actual economic dynamics. We find empirical support for the hypothesis of stochastic balanced growth in Japan. The long-run evolution of the economy may be related to two persistent shocks which bear the interpretation of productivity and labour supply shocks. Interestingly, the productivity shock explains the bulk of output fluctuations both in the short and in the long terms. The finding is coherent with real business cycle theory and provide further support for the theoretical framework employed in the paper. Coherent with the simulation results of Hayashi and Prescott (2002), we find that the potential output path has been characterised by both a reduction in level and slope. Since supply side reforms are still in their infancy, it is likely that the recovery of the Japanese economy is going to be a long lasting process.

References


21


[38] Roubini, N., 1996, Japan’s economic crisis, mimeo, Stern School of Business, New York University.


Table 1
Cointegration analysis

<table>
<thead>
<tr>
<th>Eigenvalue:</th>
<th>0.440</th>
<th>0.187</th>
<th>0.142</th>
<th>0.095</th>
<th>0.063</th>
<th>0.013</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hypothesis:</td>
<td>$r = 0$</td>
<td>$r \leq 1$</td>
<td>$r \leq 2$</td>
<td>$r \leq 3$</td>
<td>$r \leq 4$</td>
<td>$r \leq 5$</td>
</tr>
<tr>
<td>$\lambda_{\text{Trace}}$</td>
<td>200.4$^{**}$</td>
<td>96.56$^{**}$</td>
<td>59.59$^{**}$</td>
<td>32.03$^*$</td>
<td>14.08</td>
<td>2.342</td>
</tr>
<tr>
<td>95% crit. value</td>
<td>94.2</td>
<td>68.5</td>
<td>47.2</td>
<td>29.7</td>
<td>15.4</td>
<td>3.8</td>
</tr>
</tbody>
</table>

$r$ denotes the number of valid cointegrating vectors;
$^{**}$ denotes significance at the 1% level, $^*$ denotes significance at the 5% level.

Restricted cointegrating vectors
($\beta'$ matrix; cointegrating vectors normalised on $c$, $i$, $f$, and $w$ respectively)

<table>
<thead>
<tr>
<th>$y$</th>
<th>$c$</th>
<th>$i$</th>
<th>$f$</th>
<th>$w$</th>
<th>$e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-0.864$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(0.025)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-1.281$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(0.055)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-1.204$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(0.171)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-1.296$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>(0.106)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Likelihood-ratio tests: $\chi^2(4) = 3.300$ ($p$-value: 0.348)

<table>
<thead>
<tr>
<th>$y$</th>
<th>$c$</th>
<th>$i$</th>
<th>$f$</th>
<th>$w$</th>
<th>$e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-1$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$-1$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$-1$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$-1.463$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.830</td>
</tr>
<tr>
<td>(0.109)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.161)</td>
</tr>
</tbody>
</table>

Likelihood-ratio tests: $\chi^2(8) = 12.178$ ($p$-value: 0.058)
Loadings matrix
(α matrix)

\[
\begin{array}{ccccc}
& c - y & i - y & f - y & w - g(y, e) \\
y & 0.007 & -0.025 & 0.001 & -0.046 \\
(0.067) & (0.029) & (0.003) & (0.013) \\
c & -0.145 & -0.057 & -0.002 & -0.018 \\
(0.071) & (0.031) & (0.004) & (0.014) \\
i & -0.114 & -0.121 & 0.015 & -0.089 \\
(0.111) & (0.048) & (0.006) & (0.021) \\
f & -0.865 & -0.689 & -0.048 & -0.142 \\
(0.339) & (0.148) & (0.017) & (0.065) \\
w & -0.137 & -0.038 & -0.004 & -0.027 \\
(0.077) & (0.034) & (0.004) & (0.015) \\
e & 0.065 & 0.012 & 0.004 & -0.029 \\
(0.025) & (0.011) & (0.001) & (0.005) \\
\end{array}
\]

Likelihood-ratio tests: \( \chi^2(8) = 12.178 \) (p-value: 0.058)

**Table 2. The estimated common trends model**

<table>
<thead>
<tr>
<th>Variable</th>
<th>( \tau_\theta )</th>
<th>( \tau_\xi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y - e )</td>
<td>4.799 (3.482)</td>
<td></td>
</tr>
<tr>
<td>( c - e )</td>
<td>4.799 (3.482)</td>
<td></td>
</tr>
<tr>
<td>( i - e )</td>
<td>4.799 (3.482)</td>
<td></td>
</tr>
<tr>
<td>( f - e )</td>
<td>4.799 (3.482)</td>
<td></td>
</tr>
<tr>
<td>( w )</td>
<td>6.173 (4.451)</td>
<td>-0.799 (0.173)</td>
</tr>
<tr>
<td>( e )</td>
<td>1.024 (1.136)</td>
<td>0.963 (0.209)</td>
</tr>
</tbody>
</table>

\( \tau_\theta \) denotes the permanent technological shock, while \( \tau_\xi \) denotes the permanent labour supply shock. Standard errors are computed by bootstrap simulation (1000 replications).
Table 3
Forecast error variance decomposition

<table>
<thead>
<tr>
<th></th>
<th>1 year</th>
<th></th>
<th>3 years</th>
<th></th>
<th>5 years</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\tau_\theta$</td>
<td>$\tau_\xi$</td>
<td>$v$</td>
<td>$\tau_\theta$</td>
<td>$\tau_\xi$</td>
<td>$v$</td>
</tr>
<tr>
<td>$y - e$</td>
<td>0.72</td>
<td>0.10</td>
<td>0.18</td>
<td>0.82</td>
<td>0.07</td>
<td>0.11</td>
</tr>
<tr>
<td>$c - e$</td>
<td>0.66</td>
<td>0.03</td>
<td>0.31</td>
<td>0.79</td>
<td>0.01</td>
<td>0.20</td>
</tr>
<tr>
<td>$i - e$</td>
<td>0.28</td>
<td>0.42</td>
<td>0.30</td>
<td>0.51</td>
<td>0.29</td>
<td>0.20</td>
</tr>
<tr>
<td>$f - e$</td>
<td>0.03</td>
<td>0.00</td>
<td>0.97</td>
<td>0.12</td>
<td>0.01</td>
<td>0.87</td>
</tr>
<tr>
<td>$w$</td>
<td>0.00</td>
<td>0.04</td>
<td>0.96</td>
<td>0.09</td>
<td>0.13</td>
<td>0.78</td>
</tr>
<tr>
<td>$e$</td>
<td>0.15</td>
<td>0.18</td>
<td>0.67</td>
<td>0.14</td>
<td>0.31</td>
<td>0.55</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>10 years</th>
<th></th>
<th>20 years</th>
<th></th>
<th>$\infty$</th>
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<tbody>
<tr>
<td></td>
<td>$\tau_\theta$</td>
<td>$\tau_\xi$</td>
<td>$v$</td>
<td>$\tau_\theta$</td>
<td>$\tau_\xi$</td>
<td>$v$</td>
</tr>
<tr>
<td>$y - e$</td>
<td>0.95</td>
<td>0.02</td>
<td>0.03</td>
<td>0.96</td>
<td>0.01</td>
<td>0.03</td>
</tr>
<tr>
<td>$c - e$</td>
<td>0.93</td>
<td>0.01</td>
<td>0.06</td>
<td>0.97</td>
<td>0.00</td>
<td>0.03</td>
</tr>
<tr>
<td>$i - e$</td>
<td>0.86</td>
<td>0.07</td>
<td>0.07</td>
<td>0.89</td>
<td>0.05</td>
<td>0.06</td>
</tr>
<tr>
<td>$f - e$</td>
<td>0.17</td>
<td>0.12</td>
<td>0.71</td>
<td>0.14</td>
<td>0.15</td>
<td>0.71</td>
</tr>
<tr>
<td>$w$</td>
<td>0.60</td>
<td>0.18</td>
<td>0.22</td>
<td>0.84</td>
<td>0.11</td>
<td>0.05</td>
</tr>
<tr>
<td>$e$</td>
<td>0.52</td>
<td>0.37</td>
<td>0.11</td>
<td>0.44</td>
<td>0.49</td>
<td>0.07</td>
</tr>
</tbody>
</table>

$\tau_\theta$ and $\tau_\xi$ denote the permanent technological and labour supply shocks, respectively. $v$ measures the joint effect of the transitory shocks.
Figure 1: Responses to technology shock: per capita output ($y$), per capita consumption ($c$), per capita investment ($i$), per capita real stock market index ($f$), real wage ($w$), employment ($e$). Standard errors have been computed by bootstrap simulation with 1000 replications.
Figure 2: Responses to labour supply shock: per capita output (y), per capita consumption (c), per capita investment (i), per capita real stock market index (f), real wage (w), employment (e). Standard errors have been computed by bootstrap simulation with 1000 replications.
Figure 3: Actual (Yp) and smoothed (K) permanent GDP components.